The Chi-Square Test for a Population Variance

Variance Test

QBA201 – Quantitative Business Analysis

Shi Qiu – Fall 2026

When the business cares about consistency

The packaging line could have the correct mean weight and still vary too much.

Change the target

Let σ^2 be the population variance in squared grams. Compare it with a benchmark σ_0^2.

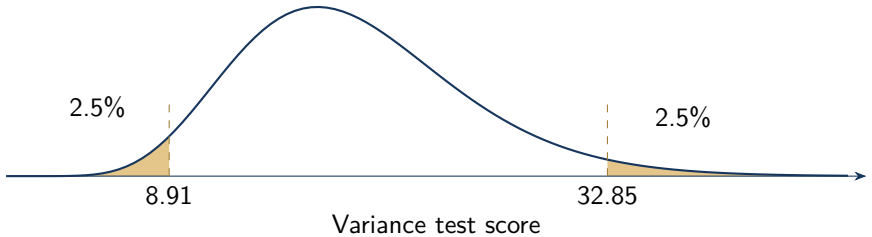
$$Q_{\text{obs}} = \frac{(n-1)s^2}{\sigma_0^2}.$$

Use the null reference

For independent normal weights, under $H_0 : \sigma^2 = \sigma_0^2$, the reference is χ^2 with $n-1$ degrees of freedom.

Equal tail areas do not mean equal distances

At 19 degrees of freedom the chi-square curve is right-skewed. For a two-sided 5% test, find the cutoffs leaving 2.5% in each tail.



For larger variance use the upper tail. For smaller variance use the lower tail. For different variance use both.

Check whether pack-weight variance has changed

An independent normal sample of 20 pack weights has $s^2 = 36$ g^2. Test whether σ^2 differs from 25 g^2 at 5%.

$$Q_{\text{obs}} = \frac{19(36)}{25} = 27.36, \qquad 8.91 < 27.36 < 32.85.$$

Read the decision

Fail to reject. The sample does not establish that variance differs from 25 g^2.

For separate preplanned higher- and lower-variance claims, the areas at this same score are 0.0965 (right) and 0.9035 (left).