The t Test for One or Two Means

Lectures 22–23

QBA201 – Quantitative Business Analysis

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What changes when we estimate the spread?

Recall the denominator

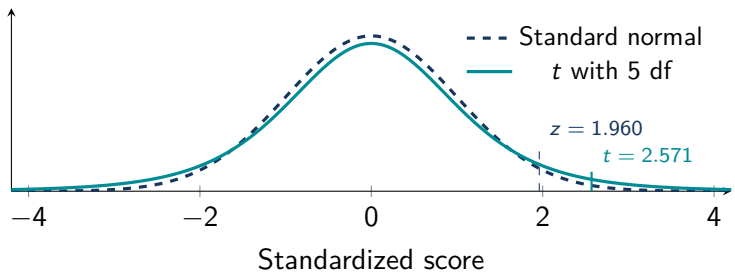
For a mean test, the numerator is the observed mean minus the benchmark: $\bar{x} - \mu_0$.

Population SD	Standard error used	Reference under H_0
Known σ	$\sigma/\sqrt{n}$	Standard normal Z
Unknown σ	$s/\sqrt{n}$	Student t with $n-1$ degrees of freedom

For independent normal observations, $\bar{X}$ stays normal. The statistic with the estimated standard error follows t.

A small sample needs a wider reference

If $n = 6$, the t reference has five degrees of freedom. Compare the upper cutoffs leaving 2.5% on their own curves.



The t tails are heavier. As degrees of freedom increase, the curve and its cutoffs approach the standard normal.

Build a t score for an average cost

An independent random sample of 25 normal parcel-handling costs gives mean AED 18.80 and sample SD AED 2.00. Test whether the population mean differs from AED 18. Use $\alpha = .05$.

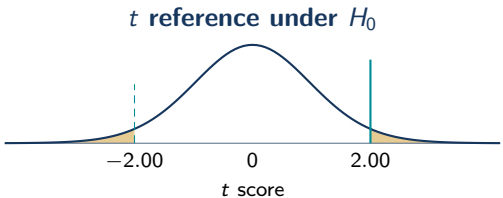
$$H_0 : \mu = 18, \qquad H_1 : \mu \neq 18.$$

Calculate the gap in estimated standard errors

$$\widehat{\mathrm{SE}} = \frac{2}{\sqrt{25}} = 0.40,$$

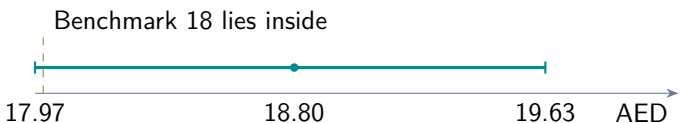
$$t_{\mathrm{obs}} = \frac{18.80 - 18}{0.40} = 2.00,$$

$$\nu = 24.$$



Check the decision against the interval

For the two-sided question, $p = 2\,\mathrm{P}(T_{24} \geq 2) = 0.0569$. At 5%, fail to reject H_0.



95% interval : $\quad 18.80 \pm 2.064(0.40) \approx [17.97, 19.63]$.

The interval includes AED 18. The data leave the difference unresolved at 5%.

Use the signed t score for a directional claim

With 24 degrees of freedom, consider three separate preplanned questions for $t_{\text{obs}} = 2.00$.

Alternative	Area under t_{24}	p-value	At 5%
$\mu > \mu_0$	Right of 2.00	.0285	Reject
$\mu < \mu_0$	Left of 2.00	.9715	Fail to reject
$\mu \neq \mu_0$	Both beyond ± 2.00	.0569	Fail to reject

A negative score of the same size reverses the one-tail areas. Use the alternative chosen before sampling.

Check whether the sampling design changes the SE

Independent normal observations

For an unknown population SD, $s/\sqrt{n}$ is the denominator for the exact one-sample t result.

Sample without replacement from a fixed list

For a simple random sample of n from a list of N units, a design-based estimated SE is

$$\frac{s}{\sqrt{n}}\sqrt{1-\frac{n}{N}}.$$

The adjustment reduces estimated variability. The independent-normal t reference needs its own assumptions.

Try a left-tail t test for delivery time

A logistics firm wants to know whether its mean delivery time is below 45 minutes. For an independent random sample of 16 delivery times from a normal population, $\bar{x} = 42.5$ and $s = 5.0$ minutes. Use $\alpha = .05$.

Choose the tail and the reference

$$H_0 : \mu \geq 45, \qquad H_1 : \mu < 45, \qquad t_{\text{obs}} = \frac{42.5 - 45}{5/\sqrt{16}} = -2.00, \qquad \nu = 15.$$

Check the business decision

The left-tail area is $\mathrm{P}(T_{15} \leq -2.00) = .0320$. Reject at 5%. The sample supports a population mean delivery time below 45 minutes.

From one mean to a difference of means

For two independent groups, $\bar{x}_i$ is the sample mean, s_i the sample SD, and n_i the sample size for $i = A, B$. The hypothesized difference is Δ_0 (zero for equal means).

Pooled t

Assumes the same population SD

$\sigma_A = \sigma_B = \sigma$ (unknown)

$$t = \frac{(\bar{x}_A - \bar{x}_B) - \Delta_0}{s_p \sqrt{\frac{1}{n_A} + \frac{1}{n_B}}}$$

$$s_p^2 = \frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2}$$

$$\mathrm{df} = n_A + n_B - 2$$

Welch t

Population SDs may differ

σ_A, σ_B (both unknown)

$$t = \frac{(\bar{x}_A - \bar{x}_B) - \Delta_0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

$$a = \frac{s_A^2}{n_A}, \qquad b = \frac{s_B^2}{n_B}$$

$$\mathrm{df} \approx \frac{(a+b)^2}{\frac{a^2}{n_A - 1} + \frac{b^2}{n_B - 1}}$$

Equal population SDs can give different sample SDs. Use z for known population SDs.

Set up a two-sample Welch t test

Two suppliers use different processes. A buyer checks whether their mean handling costs differ. Each sample is independent and random, and costs in each population are normal. The population variances are unknown and need not be equal.

Supplier	Sample size n	Sample mean (AED)	Sample SD (AED)
A	16	42.50	4.00
B	25	39.00	8.00

$$H_0 : \mu_A - \mu_B = 0, \qquad H_1 : \mu_A - \mu_B \neq 0.$$

The estimated gap is AED 3.50. We need to measure its uncertainty using both samples.

Estimate each group's contribution to uncertainty

Let $a = s_A^2/n_A$ and $b = s_B^2/n_B$ be the estimated sampling variances of the two means.

$$a = \frac{4^2}{16} = 1.00, \qquad b = \frac{8^2}{25} = 2.56.$$

Add variances, then take the square root

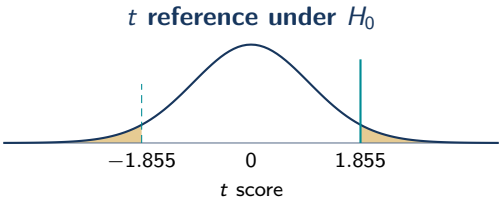
$$\widehat{\mathrm{SE}} = \sqrt{a+b} = \sqrt{3.56} \approx 1.8868, \qquad t_{\mathrm{obs}} = \frac{(42.50 - 39.00) - 0}{\sqrt{3.56}} \approx 1.8550.$$

Find the reference degrees of freedom

$$\nu \approx \frac{(a+b)^2}{a^2/(n_A - 1) + b^2/(n_B - 1)} = \frac{3.56^2}{1.00^2/15 + 2.56^2/24} \approx 37.3046.$$

Make the decision using the Welch reference

For the planned two-sided question, $t_{\text{obs}} \approx 1.8550$ with approximately 37.3046 degrees of freedom.



Planned alternative	Tail area	Decision at 5%
$\mu_A \neq \mu_B$	Both, .0715	Fail to reject
$\mu_A > \mu_B$	Right, .0358	Reject
$\mu_A < \mu_B$	Left, .9642	Fail to reject

For the buyer's two-sided question, the evidence falls short at 5%. The other rows answer different questions chosen in advance.

Pool variances when a common spread is justified

Two repair depots use the same process and equipment. Assume independent random samples from normal populations with a common unknown variance. Test whether their population mean repair times differ.

Depot	n	Sample mean (minutes)	Sample SD (minutes)
A	12	31	4
B	10	26	5

Estimate the common variance

$$s_p^2 = \frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2} = \frac{11(16) + 9(25)}{20} = 20.05.$$

Make the decision using the pooled reference

For μ_A and μ_B the population mean repair times, test $H_0 : \mu_A - \mu_B = 0$ against $H_1 : \mu_A - \mu_B \neq 0$.

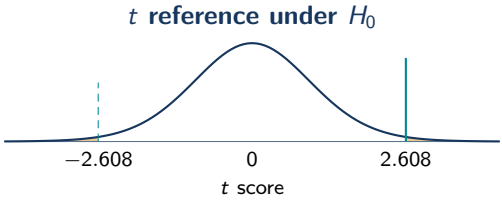
Use the common estimate

$$\widehat{\mathrm{SE}} = \sqrt{20.05(1/12 + 1/10)} \approx 1.9172,$$

$$t_{\mathrm{obs}} = \frac{(31 - 26) - 0}{\widehat{\mathrm{SE}}} \approx 2.6079.$$

Use $n_A + n_B - 2 = 20$ degrees of freedom

Two-sided $p \approx .0168$. Reject at 5%. The data support different population mean repair times.



A common variance is an assumption about the two populations. It is compatible with different means.