Probability in Words and Calculations

Lectures 8–10

QBA201 – Quantitative Business Analysis

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Three ways to assign probability

Classical

Use equally likely elementary outcomes.

Relative frequency

Use observed proportions as estimates.

Judgment

Use an assessment informed by stated evidence.

Explain where the numerical probability comes from.

Two dice: equally likely pairs, unequal sums

There are 36 equally likely ordered pairs from two independent fair dice.

$$\mathrm{P}(\text{sum } 2) = 1/36, \quad \mathrm{P}(\text{sum } 6) = 5/36, \quad \mathrm{P}(\text{sum } 10) = 3/36.$$

Discuss: Why are the 11 distinct sums not equally likely?

Observed frequencies can estimate probabilities

Desktops sold	Days
0	1
1	2
2	10
3	12
4	5
Total	30

The shop recorded daily sales over 30 days.

Use the number of days as the denominator

$$\widehat{\mathrm{P}}(\text{three sales}) = 12/30 = 0.40.$$

This describes the observed share of days with three sales.

Applying it to a future day assumes the historical period is relevant to that day's conditions.

A historical share is an estimate, not a guarantee.

Basic probability checks

Let A and B be events in a specified sample space.

$$0 \leq \mathrm{P}(A) \leq 1, \qquad \mathrm{P}(\text{all possible outcomes}) = 1.$$

If A and B cannot occur together:

$$\mathrm{P}(A \text{ or } B) = \mathrm{P}(A) + \mathrm{P}(B).$$

Discuss: What has gone wrong if a probability calculation gives 1.10?

Translate words into events

Wording	Event
Not A	Complement A^c
Both conditions	A and B
Either condition, including both	A or B
A among observations with B	$A \mid B$

Ordinary or includes the overlap unless the question says exactly one.

A complement contains everything outside the event

$$\mathrm{P}(A) + \mathrm{P}(A^c) = 1, \qquad \mathrm{P}(A^c) = 1 - \mathrm{P}(A).$$

An event and its complement exhaust the sample space without overlapping.

Discuss: If A means at least one late delivery, what does A^c mean?

Complement example: a total other than seven

Let A mean a sum of 7 on two independent fair dice.

$$\mathrm{P}(A) = 6/36 = 1/6.$$

$$\mathrm{P}(A^c) = 1 - 1/6 = 5/6.$$

The complement contains every total except 7.

Or includes the overlap

The event A or B includes:

- A without B.
- B without A.
- Both conditions together.

Count each outcome once.

Count the dice union once

Let A mean first die is 1; let B mean second die is 5. For first die 1 or second die 5:

$$6 + 6 - 1 = 11$$

distinct pairs satisfy the condition.

$$\mathrm{P}(A \text{ or } B) = 11/36.$$

Discuss: Which pair would otherwise be counted twice?

Mutually exclusive means no overlap

Here define A: total 7, and B: total 11. A single dice pair cannot satisfy both.

$$\mathrm{P}(A \text{ and } B) = 0.$$

$$\mathrm{P}(A \text{ or } B) = 6/36 + 2/36 = 8/36.$$

Disjoint events with positive probabilities are not independent.

A joint-probability table

	Outperforms B	Does not B^c	Total
Top-20 MBA A	0.11	0.29	0.40
Other MBA A^c	0.06	0.54	0.60
Total	0.17	0.83	1.00

A cell gives a row event and a column event together.

Marginal probabilities add the appropriate cells

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Total	0.17	0.83	1.00

$$\mathrm{P}(A) = 0.11 + 0.29 = 0.40, \qquad \mathrm{P}(B) = 0.11 + 0.06 = 0.17.$$

Given restricts the denominator

For $\mathrm{P}(B) > 0$:

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \text{ and } B)}{\mathrm{P}(B)}.$$

The numerator requires both conditions.

The denominator restricts attention to observations with B.

Read the condition after the vertical line first.

Translate the fund question

Find outperformance probability among managers with top-20 backgrounds.

- Desired event: B.
- Given group: A.
- Target: $\mathrm{P}(B \mid A)$.

Discuss: Which marginal total supplies the denominator?

Calculate within the given group

$$\mathrm{P}(B \mid A) = \frac{0.11}{0.40} = 0.275.$$

Within the top-20 group in this model, 27.5% outperform.

The denominator is 0.40, not the outperformance total 0.17.

Independence means an unchanged probability

When conditioning is defined:

$$\mathrm{P}(B \mid A) = \mathrm{P}(B).$$

The general product definition is:

$$\mathrm{P}(A \text{ and } B) = \mathrm{P}(A)\,\mathrm{P}(B).$$

Independence and mutual exclusivity describe different relationships.

Check independence using two equivalent comparisons

$$\mathrm{P}(B \mid A) = 0.275 \neq 0.17 = \mathrm{P}(B).$$

Equivalently:

$$\mathrm{P}(A \text{ and } B) = 0.11 \neq 0.40(0.17) = 0.068.$$

Therefore the events are dependent under this model.

Dependence alone does not establish a causal mechanism.

Translate the or question

Find the probability of outperformance or a top-20 background.

$$\text{Target: } \mathrm{P}(A \text{ or } B).$$

Include both outperformance cells, plus the top-20/no-outperformance cell.

Discuss: Which cell lies outside this event?

The addition rule gives the same result

$$\begin{aligned} \mathrm{P}(A \text{ or } B) &= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \text{ and } B). \\ &= 0.40 + 0.17 - 0.11 = 0.46. \end{aligned}$$

Discuss: Why subtract 0.11 once?

Use the complement for neither

$$(A \text{ or } B)^c = A^c \text{ and } B^c.$$

$$\mathrm{P}(A \text{ or } B) = 1 - \mathrm{P}(A^c \text{ and } B^c) = 1 - 0.54 = 0.46.$$

The complement of or is neither.

The multiplication rule does not require independence

$$\mathrm{P}(A \text{ and } B) = \mathrm{P}(A)\,\mathrm{P}(B \mid A).$$

It also equals $\mathrm{P}(B)\,\mathrm{P}(A \mid B)$.

Replace a conditional by a marginal probability only when independence is justified.

Discuss: How can a first selection change the remaining selection pool?

Without replacement: the first selection

A class has three female and seven male students. Select two distinct students at random.

Let A mean the first selection is female; B means the second selection is female.

$$\mathrm{P}(A) = 3/10.$$

We want the probability that both selections are female.

Without replacement: update the pool

Given a female first selection, two female students remain among nine students.

$$\mathrm{P}(B \mid A) = 2/9.$$

Discuss: Explain both the numerator and denominator.

Use the pool remaining after the stated condition.

Multiply along the sequence

$$\mathrm{P}(A \text{ and } B) = \frac{3}{10}\frac{2}{9} = \frac{6}{90} = 0.0667.$$

Both selections are female with probability 6.67%.

Discuss: Why is $(3/10)^2$ inappropriate for this selection procedure?

Compare the selection procedures

$$\text{Without replacement: } \frac{3}{10}\frac{2}{9} = 0.0667.$$

$$\text{Independent full-pool draws: } \frac{3}{10}\frac{3}{10} = 0.09.$$

Discuss: Which procedure lowers the probability of a second female selection after a female first selection?

Subscriptions: translate every sentence

Let S mean a Sun subscription and P a Post subscription.

$$22\% \text{ Sun: } \mathrm{P}(S) = 0.22.$$

$$35\% \text{ Post: } \mathrm{P}(P) = 0.35.$$

$$6\% \text{ both: } \mathrm{P}(S \text{ and } P) = 0.06.$$

Find $\mathrm{P}(S \text{ or } P)$.

Subscriptions: calculate and interpret

$$\mathrm{P}(S \text{ or } P) = 0.22 + 0.35 - 0.06 = 0.51.$$

$$\mathrm{P}(\text{neither}) = 1 - 0.51 = 0.49.$$

Discuss: What share subscribes to exactly one paper?

Calculate every joint probability

Path	Joint probability
Female, female	$(3/10)(2/9) = 6/90$
Female, male	$(3/10)(7/9) = 21/90$
Male, female	$(7/10)(3/9) = 21/90$
Male, male	$(7/10)(6/9) = 42/90$

Check: $(6 + 21 + 21 + 42)/90 = 1$.

The wording determines which paths count

Write F for female and M for male; list the first selection first. Exactly one female:

$$\mathrm{P}(F, M) + \mathrm{P}(M, F) = 21/90 + 21/90 = 42/90.$$

At least one female:

$$1 - \mathrm{P}(M, M) = 1 - 42/90 = 48/90.$$

Exactly one and at least one are different events.

Passing within two attempts

Suppose:

$$\text{P(pass first)} = 0.72,$$

$$\text{P(pass second} \mid \text{fail first)} = 0.88.$$

Assume all first-attempt failures try again.

Discuss: Why should 0.88 not be multiplied by 0.72?

Add disjoint routes to passing

$$\text{P(fail first and pass second)} = 0.28(0.88) = 0.2464.$$

$$\text{P(pass within two attempts)} = 0.72 + 0.2464 = 0.9664.$$

Check by complement:

$$1 - 0.28(0.12) = 0.9664.$$

Bayes' rule reverses the condition

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(B \mid A)\,\mathrm{P}(A)}{\mathrm{P}(B)}.$$

Knowing how common B is within A does not directly tell us how common A is within B.

The base rate $\mathrm{P}(A)$ matters.

A preparation-course example needs a coherent model

Let A mean a score at least 650 on the legacy example's test; B means course attendance.

$$\mathrm{P}(A) = 0.10, \quad \mathrm{P}(B \mid A) = 0.52, \quad \mathrm{P}(B \mid A^c) = 0.23.$$

Assume these probabilities refer to the same relevant population.

Translate the first conditional statement

Among high scorers, 52% attended a preparation course.

$$\mathrm{P}(B \mid A) = 0.52.$$

We want the reversed conditional:

$$\mathrm{P}(A \mid B).$$

Discuss: Which group supplies each denominator?

Translate the second conditional statement

Among those below the threshold, 23% attended a course.

$$\mathrm{P}(B \mid A^c) = 0.23.$$

This group contributes course attendees too.

The denominator for $\mathrm{P}(A \mid B)$ must include attendees from both score groups.

Build the joint-probability table

	Course B	No course B^c	Total
High score A	0.052	0.048	0.10
Other A^c	0.207	0.693	0.90
Total	0.259	0.741	1.00

For example, $\mathrm{P}(A \text{ and } B) = 0.10(0.52) = 0.052$.

Discuss: Which cell supplies the Bayes numerator?

Calculate the total probability of the evidence

$$\begin{aligned} \mathrm{P}(B) &= \mathrm{P}(A \text{ and } B) + \mathrm{P}(A^c \text{ and } B). \\ &= 0.10(0.52) + 0.90(0.23) = 0.259. \end{aligned}$$

Discuss: Why must both prior probabilities enter this calculation?

Bayes' result does not establish a course effect

$$\mathrm{P}(A \mid B) = 0.052/0.259 = 0.2008.$$

About 20.08% of attendees are high scorers in this model.

This conditional association does not establish that attendance doubles one student's probability of success.