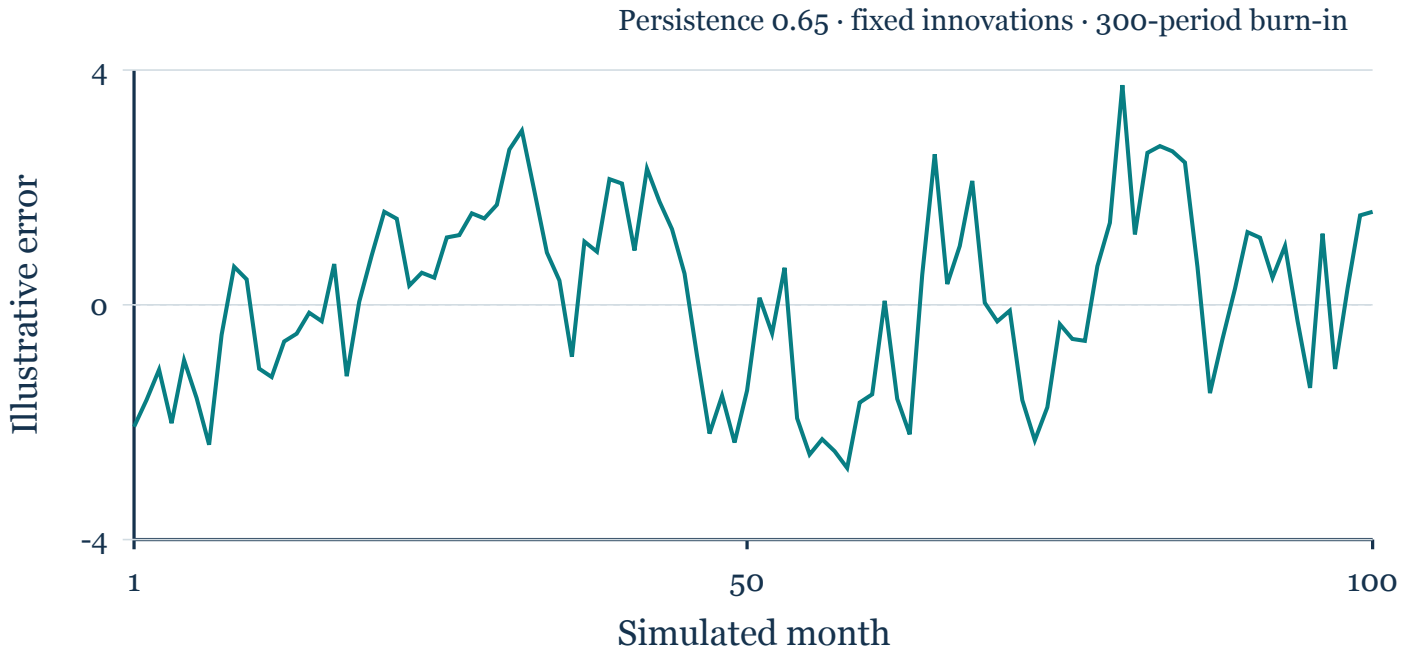
When is a persistent time series stable enough for the model?

> Distinguish persistence, stationarity and dependence.

> Then ask whether the conditional mean has used the relevant past information.

An AR(1) model carries the outcome forward

The persistence parameter determines whether shocks fade.



Illustrative simulation · fixed innovations · vertical range adapts to the selected persistence

$Y_t = a + \rho Y_{t-1} + \varepsilon_t$

One innovation with $\rho = 0.6$: $1, 0.6, 0.36, 0.216, \ldots$

Stationarity describes stable statistical features

Specify which definition is being used.

> Covariance stationary: stable finite mean/variance and lag-dependent covariance.

> AR(1) stationary regime requires $|\rho| < 1$ and suitable innovations/initialization.

> Strict stationarity concerns full joint distributions.

Strict and covariance stationarity make different distributional claims

Moment statements require the corresponding moments to exist.

> Strict: $(Y_{t_1}, \ldots, Y_{t_m})$ and $(Y_{t_1+h}, \ldots, Y_{t_m+h})$ share a distribution.

> Covariance stationary: $EY_t = \mu$, $\mathrm{Var}(Y_t) = \gamma(0) < \infty$, $\mathrm{Cov}(Y_t, Y_{t-h}) = \gamma(h)$.

> Stationarity and sufficient weak-dependence conditions are distinct.

A moving-average example makes shared shocks visible

An MA(1) observation shares one innovation with its neighbour.

> $Y_t = e_t + \theta e_{t-1}$, with iid mean-zero shocks of variance σ^2.

> $\gamma(0) = (1+\theta^2)\sigma^2$, $\gamma(1) = \theta\sigma^2$, $\gamma(h) = 0$ for $|h| \geq 2$.

> At $\theta = 0.5, \sigma^2 = 1$: $\rho(1) = 0.5/1.25 = 0.4$.

A unit root does not let shocks fade in the usual way

A random walk accumulates innovations.

> Random walk: $Y_t = Y_{t-1} + \varepsilon_t$

> If $Y_0 = 0$, $\mathrm{Var}(Y_t) = t\sigma^2$

> $\Delta Y_t = \varepsilon_t$

A permanent input change can be amplified by outcome persistence

Solve for the steady difference instead of adding coefficients blindly.

> $\Delta Y_h = 0.6\Delta Y_{h-1} + 0.2: \quad 0.20, 0.32, 0.392, \ldots$

> $\Delta Y = 0.6\Delta Y + 0.2 \Rightarrow \Delta Y = 0.5$

> Long-run multiplier: $\beta/(1-\rho)$ for a stable recursion.

Inspect persistence in the accident model

The fitted lag is a conditional association.

```
. newey logacc L.logacc trend i.month, lag(4)

------------------------------------------------------------------------------
             |             Newey–West
      logacc | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
      logacc |
         L1. |   .6783359   .0825587     8.22   0.000     .5143906    .8422811
             |
       trend |   .0009218   .0002548     3.62   0.000     .0004158    .0014278
             |
       _cons |   3.276324   .8753799     3.74   0.000     1.537993    5.014655
------------------------------------------------------------------------------
```

Calendar-indicator coefficient rows are omitted here. The output download contains the full numerical results.

Differencing changes the target relationship

It can remove an accumulating level component in a specific model.

> For the simple random walk, first differencing leaves the innovation.

> A level relationship and a relationship between changes answer different empirical questions.

Unrelated trending series can appear related

A high R-squared can be misleading.

> Two independent series with accumulating random shocks can produce a striking regression fit in levels even without an economic relationship.

> Inspect time plots and model the source of persistence before interpreting a slope.

Weak dependence concerns information far apart in time

Persistence and large-sample reasoning must be compatible.

> Weak dependence: sufficiently distant information becomes weak enough for the stated limit result.

> Stationarity alone does not guarantee that time averages behave like independent draws.

Dynamic completeness is a conditional-mean claim

Past information should not predict the remaining mean error.

> Dynamic completeness: omitted history adds no conditional-mean prediction for the stated information set.

> A lagged outcome changes both the comparison and the needed assumptions.

Define the information set before calling an error an innovation

A mean restriction is stronger than a failed residual-correlation test.

> I_t: stated current inputs and past information; $m_t = E[Y_t \mid I_t]$.

> $u_t = Y_t - m_t \Rightarrow E[u_t \mid I_t] = 0$.

> Mean unpredictability, zero autocovariance and a diagnostic test result are distinct.

Fit a dynamic conditional mean in the same monthly data

Try the comparison before opening the explanation.

Practice

Using traffic.csv, fit the model in the dynamic block of overleaf_technical.do. State the estimation dates, interpret the lagged log-accident coefficient, and explain why its estimated value does not prove stationarity or dynamic completeness. Save your own commands and robust output.

Fit a dynamic conditional mean in the same monthly data: explanation

Follow the calculation and interpret its units.

> The first lag removes the initial observation, leaving 107 months for this complete dataset. The lagged log-accident coefficient is about 0.332658 with robust SE 0.090942. Interpret it conditional on the included calendar and policy regressors. A coefficient inside minus one and one is compatible with stability of that specified AR part, but does not establish the whole data-generating process or zero conditional mean given all relevant information.

Add a lagged outcome with its timing made explicit

A different coefficient can be a different conditional comparison.

```
. newey logacc L.logacc trend i.month unem spdlaw beltlaw wkends, lag(4)

------------------------------------------------------------------------------
             |             Newey–West
      logacc | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
      logacc |
         L1. |   .3326578   .1071252     3.11   0.003     .1198023    .5455133
             |
       trend |   .0008893   .0002853     3.12   0.002     .0003223    .0014562
             |
        unem |  -.0125834   .0044189    -2.85   0.005    -.0213638   -.0038031
      spdlaw |  -.0414191   .0141509    -2.93   0.004    -.0695366   -.0133016
     beltlaw |   .0652123   .0154671     4.22   0.000     .0344794    .0959452
      wkends |   .0032099    .002814     1.14   0.257    -.0023815    .0088014
       _cons |   7.018421   1.156725     6.07   0.000     4.720033    9.316809
------------------------------------------------------------------------------
```

Calendar-indicator coefficient rows are omitted here. The output download contains the full numerical results.

From dynamic completeness to serial-correlation diagnosis

From dynamic completeness to serial-correlation diagnosis

AR long run

Try the comparison before opening the explanation.

Practice

For $Y_t = a + 0.6Y_{t-1} + 0.2X_t + e_t$, what is the long-run response to a permanent unit increase in x under a stable model?

AR long run: explanation

Follow the calculation and interpret its units.

> The steady-state change solves $\Delta Y = 0.6\Delta Y + 0.2$, so $\Delta Y = 0.2/(1 - 0.6) = 0.5$. This calculation assumes a stable dynamic relation and an exogenous permanent x change with other determinants fixed.

Trending regression

Try the comparison before opening the explanation.

Practice

Two unrelated series both rise over time and produce a high R^2 when regressed on each other. What should you do before interpreting the slope?

Trending regression: explanation

Follow the calculation and interpret its units.

> Inspect their time plots and consider deterministic trends, stochastic trends and stationarity. A high R^2 does not establish a meaningful relationship. Choose a specification and inference procedure justified by the series' properties; differencing every series automatically is also not a substitute for reasoning.