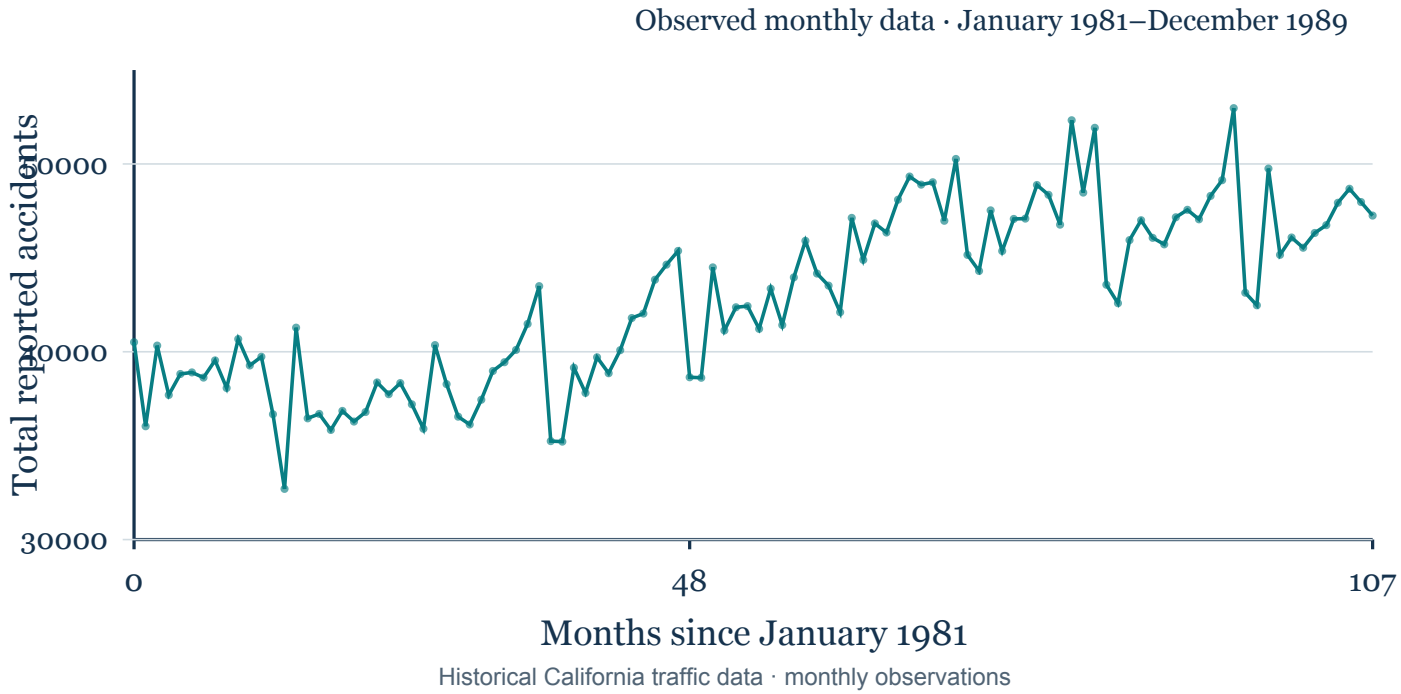
Did a traffic law change accidents, or did the calendar change?

Begin with a dated outcome and a plausible alternative explanation.



A time-only policy variable compares dates

Time controls can remove useful variation as well as confounding.

> A date-based policy contrast can mix several simultaneous changes.

> Calendar controls and an identification argument have distinct roles.

Define the monthly unit and time index

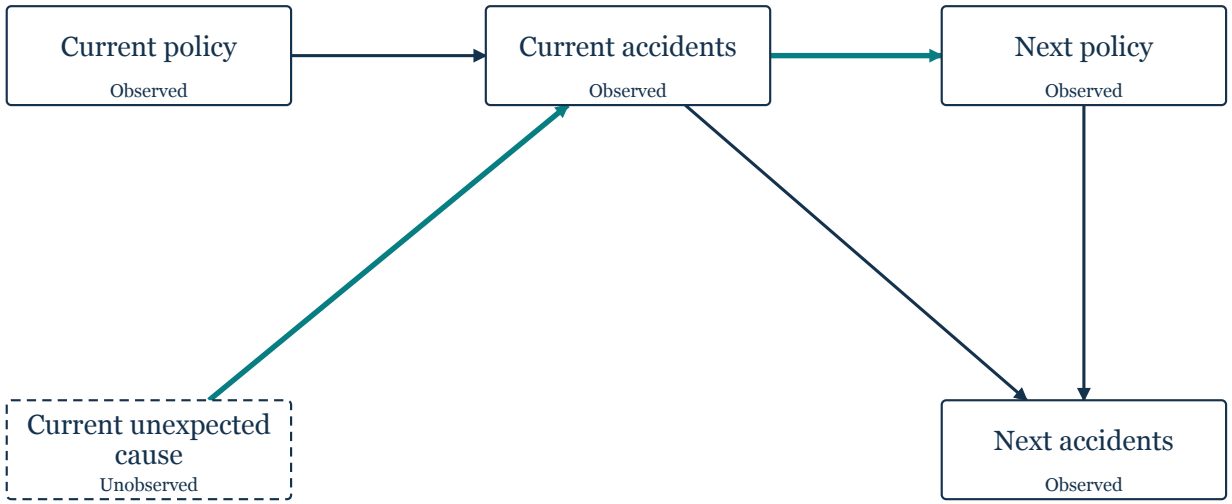
A lag requires a correct ordering, not just a row number.

> $t = 1, \ldots, 108$: monthly observations

> January 1981 through December 1989

Feedback across dates does not require a causal cycle

Current accidents can affect the policy chosen next month.



1 open directed route · 0 open alternatives · gold: conditioned on

Assumed causal structure · dashed boxes: unobserved · gold boxes: conditioned on

> $u_t \to Y_t \to X_{t+1}$

> Policy at the next date can respond to an unexpected outcome now.

Contemporaneous exogeneity concerns the current regressor vector

Current means the variables included for the same date.

> x_t: the ordered list of regressors at date t

> Contemporaneous condition: $E(u_t \mid x_t) = 0$

Strict and sequential exogeneity condition on different information

Future regressors make strict exogeneity demanding.

> Strict: $E(u_t \mid x_1, \ldots, x_T) = 0$

> Sequential: $E(u_t \mid x_1, \ldots, x_t) = 0$

Past outcomes can contain past errors

Adding dynamics changes the exogeneity question.

> A lagged outcome is the outcome from an earlier date.

> This is a reason to model timing explicitly rather than assume lagging a variable always makes it safe.

Time ordering changes the meaning of a sample

One realised history can contain strongly related observations.

> An ordered history can share information across dates.

> Justify averaging and uncertainty for the process, rather than treating rows as independent.

Finite-sample OLS assumptions apply to the complete regressor path

Temporal dependence and regressor-error dependence are different restrictions.

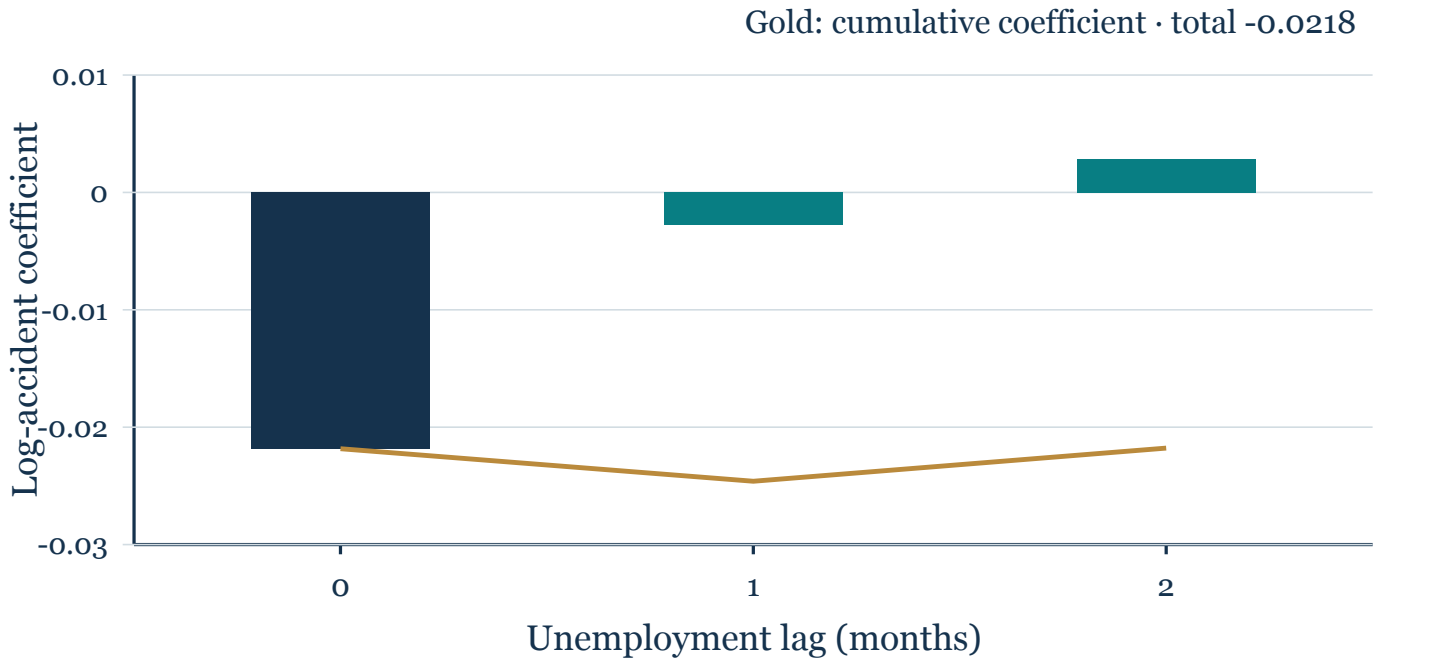
> Let $\mathcal{X} = (x_1, \ldots, x_T)$ denote the whole regressor path.

> Conditional mean: $E(u_t \mid \mathcal{X}) = 0$ for each t.

> Conventional variance: $\text{Var}(u_t \mid \mathcal{X}) = \sigma^2$ and $\text{Cov}(u_t, u_s \mid \mathcal{X}) = 0$ for $s \neq t$.

Does unemployment affect accidents only this month?

Allow a relationship to unfold over several periods.



Actual monthly unemployment coefficients from the finite distributed-lag regression. Cumulative means the sum through the selected lag.

Define lags, differences and horizons

They are different operations on ordered observations.

> $LX_t = X_{t-1}$

> $\Delta X_t = X_t - X_{t-1}$

Write a finite distributed-lag model

Every coefficient has an outcome date and a regressor date.

> $$Y_t = a + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \text{controls} + u_t$$

Trace a one-period regressor change

Its outcome responses occur at several dates.

> Horizon h: $0, 1, 2, 3$

> Temporary-input responses: $0.20, 0.10, -0.05, 0$

> Cumulative response: $0.20 + 0.10 - 0.05 = 0.25$

A permanent change has another time path

The long-run level effect reaches the lag sum in this model.

> Permanent-input responses: $0.20, 0.30, 0.25, 0.25, \ldots$

> Temporary cumulative response and permanent long-run level contrast are different quantities.

Introduce the uncertainty option before reading the lag table

Newey–West changes the covariance estimate around an unchanged OLS fit.

> Newey–West/HAC: allow suitable unequal spread and time dependence in uncertainty estimation.

> Four covariance lags here mean four months; the mean equation is unchanged.

Read the observed lag estimates

Let the application determine the pattern.

```
. newey logacc unem L(1/2).unem trend i.month, lag(4)

------------------------------------------------------------------------------
             |             Newey–West
      logacc | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        unem |
         --. |  -.0218345   .0073752    -2.96   0.004    -.0364866   -.0071823
         L1. |  -.0027711   .0075824    -0.37   0.716    -.0178349    .0122927
         L2. |   .0028242   .0079337     0.36   0.723    -.0129375    .0185858
             |
       trend |   .0018359   .0002514     7.30   0.000     .0013365    .0023352
             |
       _cons |   10.67207   .0367848   290.12   0.000     10.59899    10.74515
------------------------------------------------------------------------------
```

Calendar-indicator coefficient rows are omitted here. The output download contains the full numerical results.

Estimate uncertainty for the lag sum

The coefficient covariances matter.

```
. lincom unem+L.unem+L2.unem

------------------------------------------------------------------------------
      logacc | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         (1) |  -.0217814    .004154    -5.24   0.000    -.0300341   -.0135287
------------------------------------------------------------------------------
```

Testing no dynamic response differs from testing a zero total response

A collection of lag coefficients can cancel in its sum.

> Any included response: $H_0 : \beta_0 = \cdots = \beta_q = 0$.

> Zero long-run propensity: $H_0 : \sum_{j=0}^{q} \beta_j = 0$.

> Different null restrictions answer different empirical questions.

The sample changes when lags are added

Compare fit and tests on compatible observations.

> A two-lag model loses the first two observations if no earlier data are available.

> Define the common estimation window before comparing models or applying a fit-loss test.

Trace the actual monthly lag pattern

Reproduce the evidence, then explain its economic meaning.

Practice

Extend unemployment lags from two to three months. First restrict both fits to the same 105 available months. Compare the cumulative coefficients and their lincom SEs. Keep your commands, log, output and interpretation.

Trace the actual monthly lag pattern: explanation

A reproducible result needs both a calculation and an interpretation.

> Common 105 months: two-lag sum −0.018658; three-lag sum −0.018684.

> Corresponding robust SEs: 0.003752 and 0.003761.

> Fixing the sample isolates the change in lag specification.

Strict exogeneity

Try the comparison before opening the explanation.

Practice

Define strict and contemporaneous exogeneity for a time-series regression using x_t and error u_t. Which is stronger?

Strict exogeneity: explanation

Follow the calculation and interpret its units.

> Contemporaneous exogeneity requires $E(u_t \mid x_t) = 0$. Strict exogeneity requires $E(u_t \mid x_1, \ldots, x_T) = 0$ for every t, including future regressors. Strict implies contemporaneous, but the reverse need not hold. These statements use x_t as the entire regressor vector.

A count and a rate answer different questions

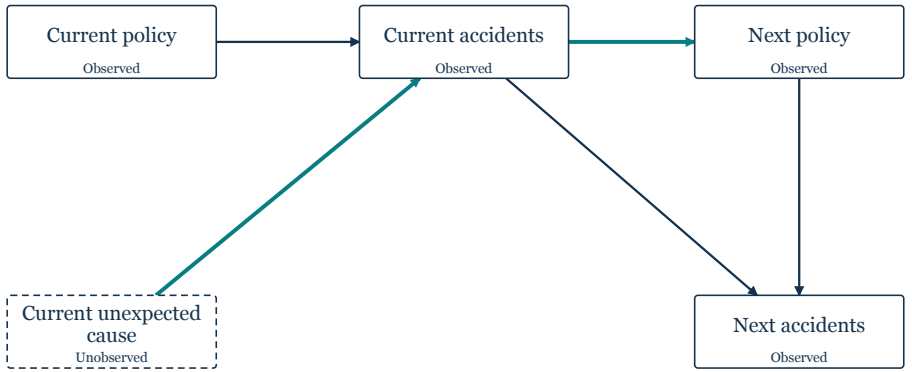
Exposure can change while individual risk stays fixed.

> Total accidents can rise because there is more driving, even if risk per trip falls.

> Our current outcome is the recorded statewide count, logged for the fitted comparison.

DAG practice: dated policy feedback

Trace the routes before choosing a control.



1 open directed route · 0 open alternatives · gold: conditioned on

Assumed causal structure · dashed boxes: unobserved · gold boxes: conditioned on

Practice

Current unexpected accident causes can affect current accidents and hence next month’s policy. Trace that dated route. Why can it challenge strict exogeneity without requiring a future policy to cause an earlier accident?

DAG practice: dated policy feedback: explanation

Explain what conditioning changes in this stated graph.

> The route is $u_t \to Y_t \to X_{t+1}$. It provides a reason that the current disturbance can be related to future regressors, which strict exogeneity conditions on. Every causal arrow still moves forward through the policy process. A sequential mean assumption uses a different information set and can be considered when current policy is chosen before the current unexpected cause, subject to the full model and timing assumptions. The graph does not prove that assumption. It makes clear why the strict version should not be imposed merely because the observations have dates.

Lag sum

Try the comparison before opening the explanation.

Practice

A finite distributed-lag model has coefficients 0.2, 0.1 and −0.05 on current x, lag 1 and lag 2. Compute the impact and the cumulative response to a one-period unit increase in x.

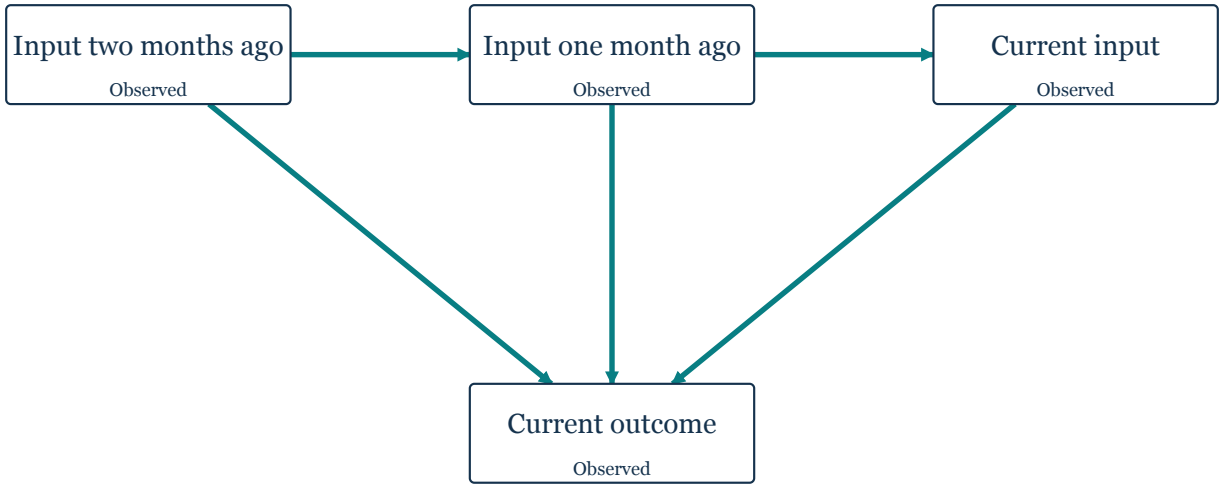
Lag sum: explanation

Follow the calculation and interpret its units.

> Impact is 0.2. The three-period cumulative response is $0.2 + 0.1 - 0.05 = 0.25$ outcome units. In this finite-lag model, a permanent unit x increase has a long-run level effect of 0.25 as well. A model also containing earlier outcomes needs a different long-run calculation, developed next.

Explain a dynamic result with a time path

One label such as effect is not enough.

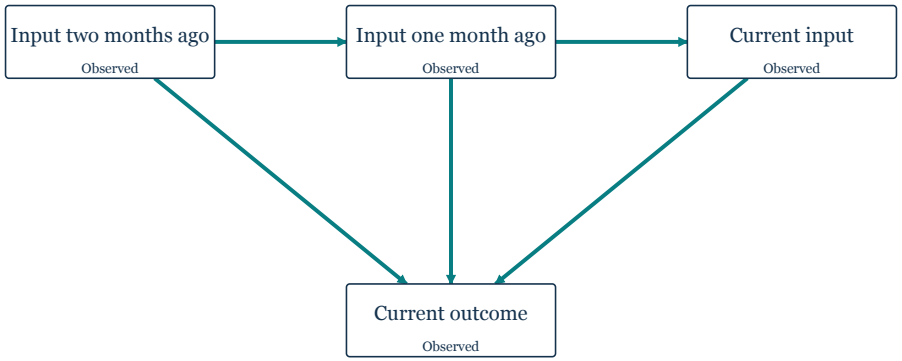


3 open directed routes · 0 open alternatives · gold: conditioned on

Assumed causal structure · dashed boxes: unobserved · gold boxes: conditioned on

DAG practice: partial lag effect or propagated input change?

Trace the routes before choosing a control.



3 open directed routes · 0 open alternatives · gold: conditioned on

Assumed causal structure · dashed boxes: unobserved · gold boxes: conditioned on

Practice

An earlier input influences the outcome directly and by changing later input values. Which routes remain if the later inputs are held fixed? Why can the response to a naturally propagated input innovation differ from the coefficient on that earlier input?

DAG practice: partial lag effect or propagated input change?: explanation

Explain what conditioning changes in this stated graph.

> Holding both later inputs fixed blocks the routes through those input nodes. The direct earlier-input-to-current-outcome route remains. Its partial contrast differs from an intervention that also changes later values of the input process. For a propagated innovation, trace the resulting entire input path and combine it with the distributed-lag coefficients. For a one-period imposed regressor increase, specify that all other dates remain fixed. A sum of lag coefficients is meaningful for the stated path and model; the word shock alone does not define which path was changed.