Binary-Model Applications – Supplement

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ECO 351 – Introduction to Econometrics

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The fitted model behind the adjusted group comparison

```
. logit deny c.pir i.black, vce(robust)
Logistic regression                                     Number of obs =  2,380
------------------------------------------------------------------------------
             |               Robust
        deny | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         pir |   5.370362   .9633435     5.57   0.000     3.482244    7.258481
     1.black |   1.272782   .1460986     8.71   0.000     .9864339     1.55913
       _cons |  -4.125558    .345825   -11.93   0.000    -4.803362   -3.447753
------------------------------------------------------------------------------
```

Let $G = 1$ for Black applicants and $G = 0$ for white applicants. The fitted index is

$$-4.125558 + 5.370362\, PIR + 1.272782\, G.$$

Apply the logistic CDF to compute probabilities. `i.black` identifies a discrete indicator comparison for `margins`.

A local slope describes the curve near one starting value

```
. margins, dydx(pir) at(pir=(0.30 0.40))
Conditional marginal effects                                    Number of obs = 2,380
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
pir          |
         _at |
          1  |   .5061048   .0687362     7.36   0.000     .3713842    .6408254
          2  |   .7131224   .1318454     5.41   0.000     .4547102    .9715347
------------------------------------------------------------------------------
```

This follows the **bivariate probit**. At PIR 0.30, a small 0.01 increase has approximate probability change

$$100(0.5061048)(0.01) \approx 0.506 \text{ percentage points.}$$

For a finite change, evaluate the two endpoint probabilities.

Average predictions over the observed PIR values

```
. margins black
Predictive margins                                      Number of obs = 2,380
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       black |
           0 |   .0942749   .0064009    14.73   0.000     .0817293    .1068204
           1 |   .2606772   .0230308    11.32   0.000     .2155377    .3058167
------------------------------------------------------------------------------
```

For each indicator setting, `margins` averages predictions across the observed applicants' PIR values.

Subtracting these two averages gives the same average discrete comparison reported by `margins, dydx(black)`.

A prediction at average PIR answers a different question

```
. margins black, atmeans
Adjusted predictions                                            Number of obs = 2,380
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       black |
          0  |   .0871477   .0063699    13.68   0.000       .074663    .0996325
          1  |   .2542282   .0240825    10.56   0.000      .2070273    .3014292
------------------------------------------------------------------------------
```

`atmeans` first sets PIR to its sample mean and evaluates both indicator settings there.

$$\Delta\widehat{p} = 0.2542282 - 0.0871477 = 0.1670805.$$

The gap at mean PIR is about 16.71 percentage points. Averaging over observed PIR gave 16.64 points. State which summary you report.

A high classification rate needs a benchmark

```
. estat classification
             -------- True --------
Classified |         D            ~D  |      Total
-----------+--------------------------+-----------
     +     |        12             6  |         18
     -     |       273          2089  |       2362
-----------+--------------------------+-----------
   Total   |       285          2095  |       2380
Sensitivity                     Pr( +| D)    4.21%
Specificity                     Pr( -|~D)   99.71%
Correctly classified                        88.28%
```

At a 0.5 cutoff, a prediction at least 0.5 is classified as denial.

Predicting approval for everyone already classifies $2095/2380 \approx 88.03\%$ correctly. Compare the overall rate with how often actual denials are identified.