Binary-Model Applications

18 Nov 2026 | Chapter 11

ECO 351 – Introduction to Econometrics

Shi Qiu – Fall 2026

Recall the comparison from last class

PIR is debt payments divided by income. The fitted probit gave:

PIR	$\widehat{p}$
0.30	0.0962
0.40	0.1570
0.50	0.2388

A report claims that every 0.10 PIR increase raises denial probability by 6.08 percentage points.

Use the $0.40 \rightarrow 0.50$ comparison to assess the claim. What does the coefficient 2.967907 change directly?

The next questions concern estimation and uncertainty

The second comparison gives

$$100(0.2388 - 0.1570) = 8.18 \text{ percentage points.}$$

The coefficient changes the index. The CDF converts that index into a probability with a slope that varies along the curve.

1. How does software choose the coefficients that define the curve?
2. How uncertain is a difference between two fitted probabilities?
3. How can we summarize a comparison after holding another variable fixed?

A model assigns probability to each observed outcome

For applicant i, let $Y_i = 1$ denote denial and p_i be the model's denial probability.

Observed outcome	Probability contribution	Example
Denied, $Y_i = 1$	p_i	$p_i = 0.7 \Rightarrow 0.7$
Approved, $Y_i = 0$	$1 - p_i$	$p_i = 0.2 \Rightarrow 0.8$

For n observations that are independent conditional on their regressors, multiply these contributions.

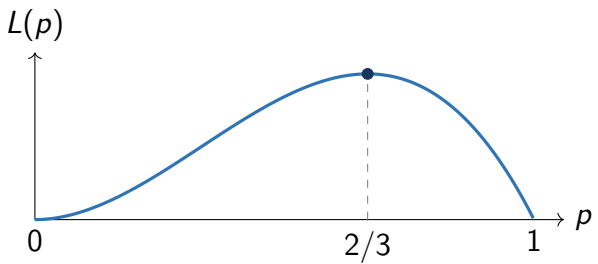
$$L = \prod_{i=1}^{n} p_i^{Y_i}(1 - p_i)^{1-Y_i}.$$

The **likelihood** treats this product as a function of the model coefficients.

See the choice of a parameter

Constructed illustration: three independent outcomes are $(1, 0, 1)$ and all share one probability p.

$$L(p) = p(1-p)p = p^2(1-p).$$



$L(0.5) = 0.125$.
$L(2/3) \approx 0.148$.
The maximum occurs at the observed fraction of ones, $\widehat{p} = 2/3$.

A regression extends this idea by making p_i depend on each applicant's covariates.

Student task: the approvals count too

Three independent applicants have outcomes **denied, approved, approved**. Two proposed models assign these denial probabilities.

	Applicant 1	Applicant 2	Applicant 3
Model A	0.7	0.2	0.2
Model B	0.8	0.4	0.4

1. Calculate the likelihood of the observed outcomes under each model.
2. Which model gives the observed sample the higher likelihood?
3. Explain why looking only at the denied applicant would give an incomplete comparison.

One set of coefficients determines the whole curve

$$L_A = 0.7(0.8)(0.8) = 0.448, \qquad L_B = 0.8(0.6)(0.6) = 0.288.$$

Model A assigns the observed triple the higher likelihood.

In probit, x_i is applicant i's PIR. All fitted probabilities follow one equation:

$$p_i = \Phi(\beta_0 + \beta_1 x_i).$$

Software varies β_0 and β_1 together, recomputes all p_i, and searches for the maximum likelihood. It commonly maximizes the log of the likelihood.

Turn the fitted model into comparison values

Return to the 2,380 mortgage applications. After fitting probit, retain the prediction estimates with `post`.

```
. margins, at(pir=(0.30 0.40 0.50)) post
Adjusted predictions                                  Number of obs = 2,380
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   .0961532   .0069017    13.93   0.000       .082626    .1096803
          2  |   .1569683   .0101166    15.52   0.000      .1371402    .1767965
          3  |   .2387884    .024485     9.75   0.000      .1907987    .2867781
------------------------------------------------------------------------------
```

Rows 1–3 set PIR to 0.30, 0.40, and 0.50. We want $\widehat{p}(0.40) - \widehat{p}(0.30)$.

Estimate the difference and its uncertainty

```
. lincom _b[2._at] - _b[1._at]
 ( 1)  - 1bn._at + 2._at = 0
------------------------------------------------------------------------------
             | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         (1) |   .0608152   .0099471     6.11   0.000     .0413193    .0803111
------------------------------------------------------------------------------
```

$$100\Delta\widehat{p} = 6.08 \text{ percentage points}, \qquad 95\% \text{ CI } \approx [4.13, 8.03] \text{ points}.$$

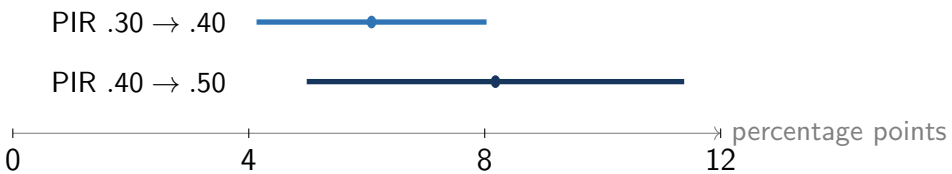
Both predictions come from the same fitted coefficients. `lincom` accounts for their shared sampling uncertainty.

Student task: report a different comparison

```
. lincom _b[3._at] - _b[2._at]
 ( 1)  - 2._at + 3._at = 0
------------------------------------------------------------------------------
             | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         (1) |   .0818201   .0163231     5.01   0.000     .0498274    .1138127
------------------------------------------------------------------------------
```

1. Which starting and ending PIR values does this compare?
2. Report the estimate and 95% interval in percentage points.
3. Write a sentence describing what this interval concerns.

Two estimates, each with its own interval



The second estimate is **8.18 percentage points**, with interval $[4.98, 11.38]$ points.

Each interval describes one contrast. Assessing whether the two contrasts differ requires a comparison of the contrasts.

An adjusted comparison needs a stated model

Keep $Y = 1$ for mortgage denial and $X = PIR$. Let $G = 1$ for Black applicants and $G = 0$ for white applicants in this teaching sample (`black`).

$$P(Y = 1 \mid X = x, G = g) = \Lambda(\beta_0 + \beta_1 x + \beta_2 g).$$

The fitted group comparison at a given PIR is

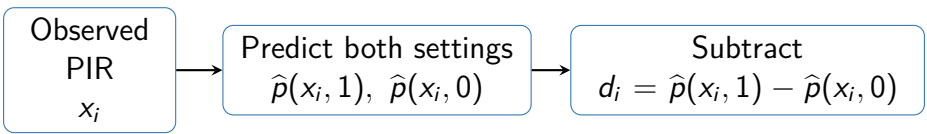
$$d(x) = \widehat{p}(x, 1) - \widehat{p}(x, 0).$$

We evaluate both settings at the **same** x.

Because the probability curve is nonlinear, $d(x)$ can vary with x. A summary must state which PIR values it uses.

Compute a comparison for each observed PIR, then average

For each observed applicant i, retain their payment-to-income ratio x_i.



For n applicants, average the n fitted differences:

$$\overline{d} = \frac{d_1 + d_2 + \cdots + d_n}{n}.$$

The indicator's **average marginal effect** is this average discrete $0 \to 1$ comparison.

Stata: continuous and indicator rows have different units

```
. margins, dydx(pir black)
Average marginal effects                                  Number of obs = 2,380
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         pir |   .5182777   .0905715     5.72   0.000     .3407609    .6957945
     1.black |   .1664023   .0240036     6.93   0.000     .1193561    .2134485
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.
```

This follows logit with `c.pir i.black`. The `1.black` row averages the discrete $0 \to 1$ comparison.

The PIR row averages the *local probability slope* per one-unit change in PIR.

Student task: interpret both rows

Use the preceding output.

1. Report the group indicator's average fitted probability gap and its 95% interval in percentage points.
2. Use the PIR row to approximate the average probability change for a **0.01** increase in PIR. Explain why this is an approximation.
3. The model adjusts for PIR. Name another applicant characteristic that could matter when interpreting the group comparison.

State the model and the comparison alongside the number.

Report the average, its scale, and the adjustment

- Group comparison: **16.64 percentage points**, with 95% interval approximately $[11.94, 21.34]$ points, adjusting for PIR.
- Small PIR increase: $100(0.5182777)(0.01) \approx \mathbf{0.52}$ percentage points, using the average local slope.
- Credit history and loan-to-value information can inform a richer comparison. Their roles need a substantive explanation.

A causal interpretation requires an identifying argument about the relevant unobserved determinants and the comparison being made.

Discussion: write the result an applied reader needs

Write a short result paragraph for the **PIR-adjusted group comparison**.

1. Identify the outcome, sample and fitted model.
2. Describe the comparison and report the estimate with its interval.
3. State one concrete limitation and one informative follow-up analysis.

Exchange paragraphs. Underline the comparison and circle the units. Ask a specific question wherever either is unclear.

A complete result and the next class

One possible report

In 2,380 Boston mortgage applications, a logit model adjusting for PIR gives an average fitted denial-probability gap of 16.64 percentage points for Black versus white applicants, with a 95% interval of 11.94–21.34 points. Credit-risk differences and the historical sample limit the interpretation.

For project sharing on **Monday, 23 November**, explain your question, data, comparison, main result and most important limitation.

Bring one informative figure and a question you would like the class to discuss.