Instrumental Variables and Two-Stage Least Squares – Supplement

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ECO 351 – Introduction to Econometrics

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Instrument exogeneity removes one covariance term

Let $Y = \ln Q$ and $X = \ln P$ in $Y = \beta_0 + \beta_1 X + u$.

$$\mathrm{Cov}(Z, Y) = \beta_1 \,\mathrm{Cov}(Z, X) + \mathrm{Cov}(Z, u)$$

$$\beta_1 = \frac{\mathrm{Cov}(Z, Y)}{\mathrm{Cov}(Z, X)}$$

The ratio requires $\mathrm{Cov}(Z, u) = 0$ and a nonzero denominator.

Weak instruments require more than routine confidence intervals

When instruments provide little independent regressor variation, 2SLS can be unstable and conventional normal-based tests can be misleading. Report first-stage evidence and consider weak-instrument-robust inference when needed.

Supplement: an additional excluded instrument

```
. ivregress 2sls logq logincome (logp = salestax cigtax), vce(robust)
Instrumental-variables 2SLS regression            Number of obs   =         48
------------------------------------------------------------------------------
             |               Robust
        logq | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
        logp |  -1.277424   .2416838    -5.29   0.000    -1.751116   -.8037325
   logincome |   .2804048   .2458276     1.14   0.254    -.2014084    .7622181
       _cons |   9.894956   .9287578    10.65   0.000     8.074624    11.71529
------------------------------------------------------------------------------
Endogenous: logp
Exogenous:  logincome salestax cigtax
```

Supplement: an overidentification test

```
. estat overid
  Test of overidentifying restrictions:
  Score chi2(1)          =  .334736  (p = 0.5629)
```

The extra restrictions are not rejected. That result does not prove both instruments valid.