Multiple Regression Model

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ECO 351 – Introduction to Econometrics

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From Quiz 2 to a controlled comparison

The unresolved issue

Quiz 2 separated a negative regression association from a causal effect. Lecture 11 showed how an omitted determinant can enter the slope.

$$E(u \mid STR) = 0 \quad \longrightarrow \quad E(u \mid STR, EL) = 0$$

Today: what does the STR coefficient mean after English-learner share is included?

What each population coefficient holds fixed

$$\mu(STR, EL) \equiv E(Score \mid STR, EL) = \beta_0 + \beta_1 STR + \beta_2 EL$$

Change *STR* only

$$\Delta\mu = \beta_1 \Delta STR, \qquad \Delta EL = 0$$

β_1: expected score change from $+1$ STR, at the same EL.

Change *EL* only

$$\Delta\mu = \beta_2 \Delta EL, \qquad \Delta STR = 0$$

β_2: expected score change from $+1$ EL percentage point, at the same STR.

A partial effect identifies what is held fixed; it does not by itself make either comparison causal.

Turn the estimated partial slopes into fitted changes

$$\widehat{Score} = 686.0 - 1.10STR - 0.650EL$$

For any two fitted comparisons,

$$\Delta\widehat{Score} = -1.10\Delta STR - 0.650\Delta EL$$

One fewer student per teacher

$\Delta STR = -1$ and $\Delta EL = 0$, so $\Delta\widehat{Score} = +1.10$ points.

Ten more EL percentage points

$\Delta EL = 10$ and $\Delta STR = 0$, so $\Delta\widehat{Score} = -6.50$ points.

State the change, the outcome units, and the variable held fixed.

Holding EL fixed means making the same substitution twice

Use $\widehat{Score} = 686 - 1.10STR - 0.650EL$ and compare districts at EL $= 10$.

$$\widehat{Score}(20, 10) = 686 - 22 - 6.5 = 657.5,$$
$$\widehat{Score}(18, 10) = 686 - 19.8 - 6.5 = 659.7.$$

The difference is 2.20 points. The intercept and EL contribution cancel.

Compare two changes at the same time

$$\widehat{Score} = 686 - 1.10STR - .650EL$$

Your task

Using the rounded equation, what fitted score change corresponds to STR falling by 1 and EL falling by 4 percentage points?

Solution: Compare two changes at the same time

$$\Delta\widehat{Score} = (-1.10)(-1) + (-0.650)(-4) = 3.70$$

The two fitted contributions add. This is a conditional prediction, not evidence that both changes could be caused by a feasible policy.

Purging means removing the part predicted by EL

First use EL to split each variable into an EL-related fitted part and a leftover residual:

$$STR_i = (\hat{a}_0 + \hat{a}_1 EL_i) + \widetilde{STR}_i,$$

$$Score_i = (\hat{c}_0 + \hat{c}_1 EL_i) + \widetilde{Score}_i.$$

Fitted part

What the district's EL share predicts for STR or Score.

Residual part

How far the district lies above or below that EL-based prediction.

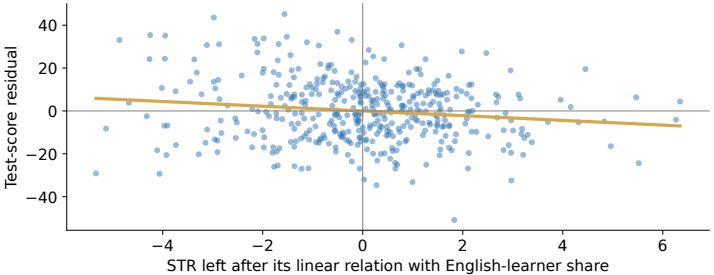
Purging subtracts fitted EL-related components; it does not delete districts or set EL equal to zero.

Purging: follow the three regressions in order

1. **Purge STR of EL:**
$$STR_i = \hat{a}_0 + \hat{a}_1 EL_i + \widetilde{STR}_i$$
2. **Purge Score of EL:**
$$Score_i = \hat{c}_0 + \hat{c}_1 EL_i + \widetilde{Score}_i$$
3. **Relate the two leftovers:**
$$\widetilde{Score}_i = \hat{\beta}_1 \widetilde{STR}_i + e_i$$

The step-3 slope equals the STR slope in $Score_i = \beta_0 + \beta_1 STR_i + \beta_2 EL_i + u_i$.

Read a point on the partial-regression plot



Horizontal residual $= +2$

Actual STR is 2 above the STR predicted from that district's EL.

Vertical residual $= -2$

Actual score is 2 below the score predicted from that district's EL.

The fitted slope through these residual pairs is $\widehat{\beta}_1 = -1.10$.

Stata: connect the full model to its partial plot

Locate the same conditional STR slope in the coefficient table and the added-variable plot.

```
do "setup.do" schools
regress score str el, vce(robust)
avplot str
lincom -str - 4*el
```

Read the result

The partial plot explains the coefficient; lincom checks the simultaneous change from the earlier exercise.

Stata output excerpt

```
. regress score str el, vce(robust)
Number of obs     =         420

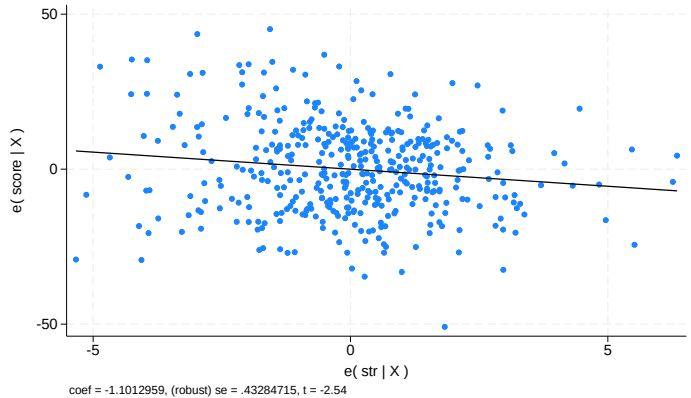
------------------------------------------------------------------------------
             |               Robust
       score | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         str |  -1.101296   .4328472    -2.54   0.011      -1.95213   -.2504616
          el |  -.6497768   .0310318   -20.94   0.000      -.710775   -.5887786
       _cons |   686.0322   8.728224    78.60   0.000     668.8754     703.189
------------------------------------------------------------------------------

. lincom -str - 4*el
 ( 1)  - str - 4*el = 0

------------------------------------------------------------------------------
       score | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         (1) |   3.700403   .4475035     8.27   0.000     2.820759    4.580047
------------------------------------------------------------------------------
```

Stata graph: partial regression for STR

Direct export from the displayed Stata graph command



The identifying assumption must include both regressors

$$E(u_i \mid STR_i, EL_i) = 0$$

After accounting for both variables, remaining score determinants must not systematically change with the included regressors. STR and EL may be correlated with each other.

A separate effect requires separate variation

If one regressor is an exact linear combination of the others, its separate coefficient cannot be determined.

Example

A model containing STR, twice STR, and an intercept cannot tell which of the two STR coefficients accounts for the same fitted movement.

Would these two controls be separately estimable?

A regression includes expenditure per student in dollars and the same expenditure in thousands of dollars, along with an intercept. Explain the problem.

Solution: Would these two controls be separately estimable?

$$Spend_{1000} = Spend_{dollars}/1000$$

The second measure adds no independent variation. Only a combination of the two coefficients is identified; omit one redundant representation.

More regressors also change the fit calculation

With k regressors plus an intercept, OLS fits $k+1$ coefficients.

$$SER = \sqrt{SSR/(n-k-1)}, \qquad \bar{R}^2 = 1 - \frac{SSR/(n-k-1)}{TSS/(n-1)}$$

Ordinary R^2 cannot fall when a regressor is added on the same sample. Adjusted R^2 can fall because it accounts for the extra coefficient.

Fit and causal credibility answer different questions

Fit

On the same sample, adding a regressor cannot lower ordinary R^2. Adjusted R^2 may fall because it accounts for the additional coefficient.

Identification

A higher fit measure does not show that the added variable is a credible control or that the remaining error satisfies zero conditional mean.

How multiple regression changes the comparison

Conclusion

A multiple-regression slope describes a change holding included regressors fixed. Partialling out shows which remaining variation is used; the expanded conditional-mean assumption still needs justification.

What does zero STR residual mean in the partial-regression plot?